Week 9 Worksheet Tuesday

Instructions. Follow the instructions given by your TA. You are not expected to finish all the problems. :)

Topics: 1.Derivative of Inverse Trig
2. Related Rates
3. Intermediate Value Theorem (IVT)
4. Warm up for Graphing.

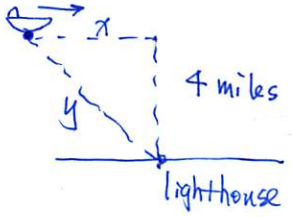
$\frac{d[\arctan x]}{dx} = \frac{1}{1+x^2}$

$\frac{d[\arcsin x]}{dx} = \frac{1}{\sqrt{1-x^2}}$

1. $y = \tan(1-x^2)\arctan(\sqrt{x+1})$ $\frac{dy}{dx} = ?$

$\frac{dy}{dx} = \sec^2(1-x^2)\cdot(-2x)\arctan(\sqrt{x+1}) + \tan(1-x^2)\frac{1}{1+x+1}\cdot\frac{1}{2}(x+1)^{-\frac{1}{2}}$

2. *(from 2016 exam)* A boat is moving parallel to a straight coast line, at a distance of 4 miles from the shore. The boat is moving at a speed of 12 miles/hr. Further along the shore there is a lighthouse. At what rate is the distance between the boat and the lighthouse changing, at the moment when the boat is precisely 5 miles away from the lighthouse?



x = horizontal distance of the boat from the light house.

y = distance between the boat and the light house.

$\frac{dx}{dt} = 12$ $y = 5$ $\frac{dy}{dx} = ?$

Equation: $x^2 + 4^2 = y^2$

$\Rightarrow 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

x: $\frac{dx}{dt} = 12$; $y = 5$; $\frac{dy}{dt} = ?$

Plug in back to equation.

$x^2+4^2 = 5^2 \Rightarrow x = 3$

$\Rightarrow 2\cdot 3\cdot 12 = 2\cdot 5\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{36}{5}$ miles/hr

3. Show that $2x + \sin x = \pi$ has a solution on the interval $[0, \pi]$. Is it the only solution in the interval $[0, \pi]$?

IVT

(a) $f(x) = 2x + \sin x$ is continuous on $[0, \pi]$

$f(0) = 0 < \pi \quad f(\pi) = 2\pi > \pi$

By IVT, there exists a number $c \in [0, \pi]$ s.t. $f(c) = \pi$.

(b) Yes! Notice $f(x)$ is increasing in $[0, \pi]$, because

$f'(x) = 2 + \cos x > 0$.

4. Consider the function

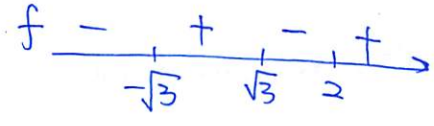
$$f(x) = \frac{x^2 - 3}{x - 2}$$

(a) Determine on which intervals the function is positive or negative.

(b) Determine on which intervals the function increases or decreases.

(c) Find all asymptotes.

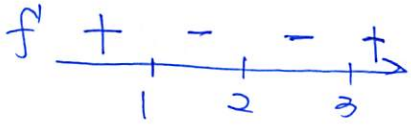
(a) $f(x) = \dfrac{(x-\sqrt{3})(x+\sqrt{3})}{x-2}$



positive: $(-\sqrt{3}, \sqrt{3})$, $(2, +\infty)$

negative: $(-\infty, -\sqrt{3})$, $(\sqrt{3}, 2)$

(b) $f'(x) = \dfrac{2x(x-2) - (x^2-3)}{(x-2)^2} = \dfrac{x^2 - 4x + 3}{(x-2)^2} = \dfrac{(x-1)(x-3)}{(x-2)^2}$



increase: $(3, +\infty)$, $(-\infty, 1)$

decrease: $(1, 3)$

(c) VA. $\boxed{x = 2}$

HA. None. (top is 1 degree higher than bottom)

SA. $m = \lim_{x \to \infty} \dfrac{f(x)}{x} = \lim_{x \to \infty} \dfrac{x^2 - 3}{x^2 - 2x} = 1$

$b = \lim_{x \to \infty} f(x) - mx = \lim_{x \to \infty} \dfrac{x^2 - 3}{x - 2} - x = \lim_{x \to \infty} \dfrac{x^2 - 3 - x^2 + 2x}{x - 2} = \lim_{x \to \infty} \dfrac{2x - 3}{x - 2} = 2.$

$\boxed{y = x + 2.}$